

Chapter 11

Gravity

Problems: 2, 5, 7, 11, 32, 36, 47, 48, 50, 73, 98

Think about: 8, 25, 39

2 • If the mass of a small Earth-orbiting satellite is doubled, the radius of its orbit can remain constant if the speed of the satellite (a) increases by a factor of 8, (b) increases by a factor of 2, (c) does not change, (d) is reduced by a factor of 8, (e) is reduced by a factor of 2.

Determine the Concept We can apply Newton's 2nd law and the law of gravity to the satellite to obtain an expression for its speed as a function of the radius of its orbit.

Apply Newton's 2nd law to the satellite to obtain:

$$\sum F_{\text{radial}} = \frac{GMm}{r^2} = m \frac{v^2}{r}$$

where M is the mass of the object the satellite is orbiting and m is the mass of the satellite.

Solving for v yields:

$$v = \sqrt{\frac{GM}{r}}$$

Thus the speed of the satellite is independent of its mass and (c) is correct.

5 • Venus has no natural satellites. However artificial satellites have been placed in orbit around it. To use one of their orbits to determine the mass of Venus, what orbital parameters would you have to measure? How would you then use them to do the mass calculation?

Determine the Concept To obtain the mass M of Venus you need to measure the period T and semi-major axis a of the orbit of one of the satellites, substitute the measured values into $T^2/a^3 = 4\pi^2/(GM)$ (Kepler's 3rd law), and solve for M .

7 • [SSM] At the surface of the moon, the acceleration due to the gravity of the moon is a . At a distance from the center of the moon equal to four times the radius of the moon, the acceleration due to the gravity of the moon is (a) $16a$, (b) $a/4$, (c) $a/3$, (d) $a/16$, (e) None of the above.

Picture the Problem The acceleration due to gravity varies inversely with the square of the distance from the center of the moon.

Express the dependence of the acceleration due to the gravity of the moon on the distance from its center:

$$a' \propto \frac{1}{r^2}$$

Express the dependence of the acceleration due to the gravity of the moon at its surface on its radius:

$$a \propto \frac{1}{R_M^2}$$

Divide the first of these expressions by the second to obtain:

$$\frac{a'}{a} = \frac{R_M^2}{r^2}$$

Solving for a' and simplifying yields:

$$a' = \frac{R_M^2}{r^2} a = \frac{R_M^2}{(4R_M)^2} a = \frac{1}{16} a$$

and $\boxed{(d)}$ is correct.

8 • At a depth equal to half the radius of Earth, the acceleration due to gravity is about (a) g (b) $2g$ (c) $g/2$, (d) $g/4$, (e) $g/8$, (f) You cannot determine the answer based on the data given.

Picture the Problem We can use Newton's law of gravity and the assumption of uniform density to express the ratio of the acceleration due to gravity at a depth equal to half the radius of Earth to the acceleration due to gravity at the surface of Earth.

The acceleration due to gravity at a depth equal to half the radius of Earth is given by:

$$g_{\frac{1}{2}r} = \frac{GM'}{\left(\frac{1}{2}r\right)^2} = \frac{4GM'}{r^2}$$

where M' is the mass of Earth between the location of interest and the center of Earth.

The acceleration due to gravity at the surface of Earth is given by:

$$g = \frac{GM}{r^2}$$

Dividing the first of these equations by the second and simplifying yields:

$$\frac{g_{\frac{1}{2}r}}{g} = \frac{\frac{4GM'}{r^2}}{\frac{GM}{r^2}} = \frac{4M'}{M} \quad (1)$$

Express M' in terms of the average density of Earth ρ and the volume V' of Earth between the location of interest and the center of Earth:

$$M' = \rho V' = \rho \left[\frac{4}{3} \pi \left(\frac{1}{2} r \right)^3 \right] = \frac{1}{6} \pi \rho r^3$$

Express M in terms of the average density of Earth ρ and the volume V of Earth:

$$M = \rho V = \rho \left(\frac{4}{3} \pi r^3 \right) = \frac{4}{3} \pi \rho r^3$$

Substitute for M and M' in equation (1) and simplify to obtain:

$$\frac{g_{\frac{1}{2}r}}{g} = \frac{4 \left(\frac{1}{6} \pi \rho r^3 \right)}{\frac{4}{3} \pi \rho r^3} = \frac{1}{2}$$

and $\boxed{(c)}$ is correct.

11 •• [SSM] Suppose the escape speed from a planet was only slightly larger than the escape speed from Earth, yet it was considerably larger than Earth. How would the planet's (average) density compare to Earth's (average) density? (a) It must be more dense. (b) It must be less dense. (c) It must be the same density. (d) You cannot determine the answer based on the data given.

Picture the Problem The densities of the planets are related to the escape speeds from their surfaces through $v_e = \sqrt{2GM/R}$.

The escape speed from the planet is given by:

$$v_{\text{planet}} = \sqrt{\frac{2GM_{\text{planet}}}{R_{\text{planet}}}}$$

The escape speed from Earth is given by:

$$v_{\text{Earth}} = \sqrt{\frac{2GM_{\text{Earth}}}{R_{\text{Earth}}}}$$

Expressing the ratio of the escape speed from the planet to the escape speed from Earth and simplifying yields:

$$\frac{v_{\text{planet}}}{v_{\text{Earth}}} = \frac{\sqrt{\frac{2GM_{\text{planet}}}{R_{\text{planet}}}}}{\sqrt{\frac{2GM_{\text{Earth}}}{R_{\text{Earth}}}}} = \sqrt{\frac{R_{\text{Earth}}}{R_{\text{planet}}} \frac{M_{\text{planet}}}{M_{\text{Earth}}}}$$

Because $v_{\text{planet}} \approx v_{\text{Earth}}$:

$$1 \approx \sqrt{\frac{R_{\text{Earth}}}{R_{\text{planet}}} \frac{M_{\text{planet}}}{M_{\text{Earth}}}}$$

Squaring both sides of the equation yields:

$$1 \approx \frac{R_{\text{Earth}}}{R_{\text{planet}}} \frac{M_{\text{planet}}}{M_{\text{Earth}}}$$

Express M_{planet} and M_{Earth} in terms of their densities and simplify to obtain:

$$1 \approx \frac{R_{\text{Earth}} \rho_{\text{planet}} V_{\text{planet}}}{R_{\text{planet}} \rho_{\text{Earth}} V_{\text{Earth}}} = \frac{R_{\text{Earth}} \rho_{\text{planet}} V_{\text{planet}}}{R_{\text{planet}} \rho_{\text{Earth}} V_{\text{Earth}}} = \frac{R_{\text{Earth}} \rho_{\text{planet}} \frac{4}{3} \pi R_{\text{planet}}^3}{R_{\text{planet}} \rho_{\text{Earth}} \frac{4}{3} \pi R_{\text{Earth}}^3} = \frac{\rho_{\text{planet}} R_{\text{planet}}^2}{\rho_{\text{Earth}} R_{\text{Earth}}^2}$$

Solving for the ratio of the densities yields:

$$\frac{\rho_{\text{planet}}}{\rho_{\text{Earth}}} \approx \frac{R_{\text{Earth}}^2}{R_{\text{planet}}^2}$$

Because the planet is considerably larger than Earth:

$$\frac{\rho_{\text{planet}}}{\rho_{\text{Earth}}} \ll 1$$

and (b) is correct.

25 •• [SSM] One of the so-called "Kirkwood gaps" in the asteroid belt occurs at an orbital radius at which the period of the orbit is half that of Jupiter's. The reason there is a gap for orbits of this radius is because of the periodic pulling (by Jupiter) that an asteroid experiences at the same place in its orbit every *other* orbit around the sun. Repeated tugs from Jupiter of this kind would eventually change the orbit of such an asteroid – therefore all asteroids that would otherwise have orbited at this radius have presumably been cleared away from the area due to this resonance phenomenon. How far from the sun is this particular 2:1 "Kirkwood" gap?

Picture the Problem The period of an orbit is related to its semi-major axis (for circular orbits this distance is the orbital radius). Because we know the orbital periods of Jupiter and a hypothetical asteroid in the Kirkwood gap, we can use Kepler's 3rd law to set up a proportion relating the orbital periods and average distances of Jupiter and the asteroid from the Sun from which we can obtain an expression for the orbital radius of an asteroid in the Kirkwood gap.

Use Kepler's 3rd law to relate Jupiter's orbital period to its mean distance from the Sun:

$$T_{\text{Jupiter}}^2 = Cr_{\text{Jupiter}}^3$$

Use Kepler's 3rd law to relate the orbital period of an asteroid in the Kirkwood gap to its mean distance from the Sun:

$$T_{\text{Kirkwood}}^2 = Cr_{\text{Kirkwood}}^3$$

Dividing the second of these equations by the first yields:

$$\frac{T_{\text{Kirkwood}}^2}{T_{\text{Jupiter}}^2} = \frac{Cr_{\text{Kirkwood}}^3}{Cr_{\text{Jupiter}}^3} = \frac{r_{\text{Kirkwood}}^3}{r_{\text{Jupiter}}^3}$$

Solving for r_{Kirkwood} yields:

$$r_{\text{Kirkwood}} = \sqrt[3]{\left(\frac{T_{\text{Kirkwood}}}{T_{\text{Jupiter}}}\right)^2} r_{\text{Jupiter}}$$

Because the period of the orbit of an asteroid in the Kirkwood gap is half that of Jupiter's:

$$\begin{aligned} r_{\text{Kirkwood}} &= \sqrt[3]{\left(\frac{\frac{1}{2} T_{\text{Jupiter}}}{T_{\text{Jupiter}}}\right)^2} (77.8 \times 10^{10} \text{ m}) \\ &= \boxed{4.90 \times 10^{11} \text{ m}} \end{aligned}$$

32 • Some people think that shuttle astronauts are "weightless" because they are "beyond the pull of Earth's gravity." In fact, this is completely untrue. (a) What is the magnitude of the gravitational field in the vicinity of a shuttle orbit? A shuttle orbit is about 400 km above the ground. (b) Given the answer in Part (a), explain why shuttle astronauts do suffer from adverse biological affects such as muscle atrophy even though they are actually not "weightless"?

Determine the Concept The weight of anything, including astronauts, is the reading of a scale from which the object is suspended or on which it rests. That is, it is the magnitude of the normal force acting on the object. If the scale reads zero, then we say the object is "weightless." The pull of Earth's gravity, on the other hand, depends on the local value of the acceleration of gravity and we can use Newton's law of gravity to find this acceleration at the elevation of the shuttle.

(a) Apply Newton's law of gravitation to an astronaut of mass m in a shuttle at a distance h above the surface of Earth:

$$mg_{\text{shuttle}} = \frac{GmM_E}{(h + R_E)^2}$$

Solving for g_{shuttle} yields:

$$g_{\text{shuttle}} = \frac{GM_E}{(h + R_E)^2}$$

Substitute numerical values and evaluate g_{shuttle} :

$$g_{\text{shuttle}} = \frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(400 \text{ km} + 6370 \text{ km})^2} = \boxed{8.71 \text{ m/s}^2}$$

(b) In orbit, the astronauts experience only one (the gravitational force) of the two forces (the second being the normal force – a compressive force – exerted by Earth) that normally acts on them. Lacking this compressive force, their bones and muscles, the absence of an exercise program, will weaken. In orbit the astronauts are not weightless, they are normal-forceless.

36 • Suppose that Earth retained its present mass but was somehow compressed to half its present radius. What would be the value of g at the surface of this new, compact planet?

Picture the Problem We can relate the acceleration due to gravity of a test object at the surface of the new planet to the acceleration due to gravity at the surface of Earth through use of the law of gravity and Newton's 2nd law of motion.

Letting a represent the acceleration due to gravity at the surface of this new planet and m the mass of a test object, apply Newton's 2nd law and the law of gravity to obtain:

$$\sum F_{\text{radial}} = \frac{GmM_E}{\left(\frac{1}{2}R_E\right)^2} = ma \Rightarrow a = \frac{GM_E}{\left(\frac{1}{2}R_E\right)^2}$$

Simplify this expression to obtain:

$$a = 4\left(\frac{GM_E}{R_E^2}\right) = 4g = \boxed{39.2 \text{ m/s}^2}$$

39 •• The speed of an asteroid is 20 km/s at perihelion and 14 km/s at aphelion. (a) Determine the ratio of the aphelion to perihelion distances. (b) Is this asteroid farther from the Sun or closer to the Sun than Earth, on average? Explain.

Picture the Problem We can use conservation of angular momentum to relate the asteroid's aphelion and perihelion distances.

(a) Using conservation of angular momentum, relate the angular momenta of the asteroid at aphelion and perihelion:

$$L_a - L_p = 0$$

or

$$mv_a r_a - mv_p r_p = 0 \Rightarrow \frac{r_a}{r_p} = \frac{v_p}{v_a}$$

Substitute numerical values and evaluate the ratio of the asteroid's aphelion and perihelion distances:

$$\frac{r_a}{r_p} = \frac{20 \text{ km/s}}{14 \text{ km/s}} = \boxed{1.4}$$

(b) It is farther from the Sun than Earth. Kepler's third law ($T^2 = Cr_{\text{av}}^3$) tells us that longer orbital periods together with larger orbital radii means slower orbital

speeds, so the speed of objects orbiting the Sun decreases with distance from the Sun. The average orbital speed of Earth, given by $v = 2\pi r_{\text{ES}}/T_{\text{ES}}$, is approximately 30 km/s. Because the given maximum speed of the asteroid is only 20 km/s, the asteroid is further from the Sun.

42 •• Suppose that the attractive interaction between a star of mass M and a planet of mass $m \ll M$ is of the form $F = KMm/r$, where K is the gravitational constant. What would be the relation between the radius of the planet's circular orbit and its period?

Picture the Problem We can use the law of gravity and Newton's 2nd law to relate the force exerted on the planet by the star to its orbital speed and the definition of the period to relate it to the radius of the orbit.

The period of the planet is related to its orbital speed:

$$T = \frac{2\pi r}{v} \quad (1)$$

Using the law of gravity and Newton's 2nd law, relate the force exerted on the planet by the star to its centripetal acceleration:

$$F_{\text{net}} = \frac{KMm}{r} = m \frac{v^2}{r} \Rightarrow v = \sqrt{KM}$$

Substitute for v in equation (1) to obtain:

$$T = \frac{2\pi}{\sqrt{KM}} r$$

43 •• [SSM] Earth's radius is 6370 km and the moon's radius is 1738 km. The acceleration of gravity at the surface of the moon is 1.62 m/s². What is the ratio of the average density of the moon to that of Earth?

Picture the Problem We can use the definitions of the gravitational fields at the surfaces of Earth and the moon to express the accelerations due to gravity at these locations in terms of the average densities of Earth and the moon. Expressing the ratio of these accelerations will lead us to the ratio of the densities.

Express the acceleration due to gravity at the surface of Earth in terms of Earth's average density:

$$\begin{aligned} g_{\text{E}} &= \frac{GM_{\text{E}}}{R_{\text{E}}^2} = \frac{G\rho_{\text{E}}V_{\text{E}}}{R_{\text{E}}^2} = \frac{G\rho_{\text{E}}\frac{4}{3}\pi R_{\text{E}}^3}{R_{\text{E}}^2} \\ &= \frac{4}{3}G\rho_{\text{E}}\pi R_{\text{E}} \end{aligned}$$

The acceleration due to gravity at the surface of the moon in terms of the moon's average density is:

$$g_{\text{M}} = \frac{4}{3}G\rho_{\text{M}}\pi R_{\text{M}}$$

Divide the second of these equations by the first to obtain:

$$\frac{g_M}{g_E} = \frac{\rho_M R_M}{\rho_E R_E} \Rightarrow \frac{\rho_M}{\rho_E} = \frac{g_M R_E}{g_E R_M}$$

Substitute numerical values and evaluate $\frac{\rho_M}{\rho_E}$:

$$\frac{\rho_M}{\rho_E} = \frac{(1.62 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})}{(9.81 \text{ m/s}^2)(1.738 \times 10^6 \text{ m})} = \boxed{0.605}$$

44 • The weight of a standard object defined as having a mass of exactly 1.00... kg is measured to be 9.81 N. In the same laboratory, a second object weighs 56.6 N. (a) What is the mass of the second object? (b) Is the mass you determined in Part (a) gravitational or inertial mass?

Picture the Problem Newton's 2nd law of motion relates the weights of these two objects to their masses and the acceleration due to gravity.

(a) Apply Newton's 2nd law to the standard object:

$$F_{\text{net}} = w_1 = m_1 g$$

Apply Newton's 2nd law to the object of unknown mass:

$$F_{\text{net}} = w_2 = m_2 g$$

Eliminate g between these two equations and solve for m_2 :

$$m_2 = \frac{w_2}{w_1} m_1$$

Substitute numerical values and evaluate m_2 :

$$m_2 = \frac{56.6 \text{ N}}{9.81 \text{ N}} (1.00 \text{ kg}) = \boxed{5.77 \text{ kg}}$$

(b) Because this result is determined by the effect on m_2 of Earth's gravitational field, it is the *gravitational* mass of the second object.

47 • [SSM] Find the escape speed for a projectile leaving the surface of the moon. The acceleration of gravity on the moon is 0.166 times that on Earth and the moon's radius is $0.273 R_E$.

Picture the Problem The escape speed from the moon or Earth is given by $v_e = \sqrt{2GM/R}$, where M and R represent the masses and radii of the moon or Earth.

Express the escape speed from the moon:

$$v_{e,m} = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{2g_m R_m} \quad (1)$$

Express the escape speed from Earth:

$$v_{e,E} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2g_E R_E} \quad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{v_{e,m}}{v_{e,E}} = \frac{\sqrt{g_m R_m}}{\sqrt{g_E R_E}} = \sqrt{\frac{g_m R_m}{g_E R_E}}$$

Solving for $v_{e,m}$ yields:

$$v_{e,m} = \sqrt{\frac{g_m R_m}{g_E R_E}} v_{e,E}$$

Substitute numerical values and evaluate $v_{e,m}$:

$$\begin{aligned} v_{e,m} &= \sqrt{(0.166)(0.273)}(11.2 \text{ km/s}) \\ &= \boxed{2.38 \text{ km/s}} \end{aligned}$$

48 •• What initial speed would a particle have to be given at the surface of Earth if it is to have a final speed that is equal to its escape speed when it is very far from Earth? Neglect any effects due to air resistance.

Picture the Problem Let the zero of gravitational potential energy be at infinity, m represent the mass of the particle, and the subscript E refer to Earth. When the particle is very far from Earth, the gravitational potential energy of the Earth-particle system is zero. We'll use conservation of energy to relate the initial potential and kinetic energies of the particle-Earth system to the final kinetic energy of the particle.

Use conservation of energy to relate the initial energy of the system to its energy when the particle is very far away:

$$\begin{aligned} K_f - K_i + U_f - U_i &= 0 \\ \text{or, because } U_f &= 0, \\ K(\infty) - K(R_E) - U(R_E) &= 0 \end{aligned} \quad (1)$$

Substitute in equation (1) to obtain:

$$\begin{aligned} \frac{1}{2}mv_\infty^2 - \frac{1}{2}mv_i^2 + \frac{GM_E m}{R_E} &= 0 \\ \text{or, because } GM_E &= gR_E^2, \\ \frac{1}{2}mv_\infty^2 - \frac{1}{2}mv_i^2 + mgR_E &= 0 \end{aligned}$$

Solving for v_i yields:

$$v_i = \sqrt{v_\infty^2 + 2gR_E}$$

Substitute numerical values and evaluate v_i :

$$v_i = \sqrt{(11.2 \times 10^3 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})} = \boxed{15.8 \text{ km/s}}$$

50 •• The science fiction writer Robert Heinlein once said, "If you can get into orbit, then you're halfway to anywhere." Justify this statement by comparing the minimum energy needed to place a satellite into low Earth orbit ($h = 400 \text{ km}$) to that needed to set it completely free from the bonds of Earth's gravity. Neglect any effects of air resistance.

Picture the Problem We'll consider a rocket of mass m which is initially on the surface of Earth (mass M and radius R) and compare the kinetic energy needed to get the rocket to its escape speed with its kinetic energy in a low circular orbit around Earth. We can use conservation of energy to find the escape kinetic energy and Newton's law of gravity to derive an expression for the low-Earth orbit kinetic energy.

Apply conservation of energy to relate the initial energy of the rocket to its escape kinetic energy:

$$K_f - K_i + U_f - U_i = 0$$

Letting the zero of gravitational potential energy be at infinity we have $U_f = K_f = 0$ and:

$$-K_i - U_i = 0$$

or

$$K_e = -U_i = \frac{GMm}{R}$$

Apply Newton's law of gravity to the rocket in orbit at the surface of Earth to obtain:

$$\frac{GMm}{R^2} = m \frac{v^2}{R}$$

Rewrite this equation to express the low-Earth orbit kinetic energy K_o of the rocket:

$$K_o = \frac{1}{2}mv^2 = \frac{GMm}{2R}$$

Express the ratio of K_o to K_e and simplify to obtain:

$$\frac{K_o}{K_e} = \frac{\frac{GMm}{2R}}{\frac{GMm}{R}} = \frac{1}{2}$$

Solving for K_e yields:

$$K_e = \boxed{2K_o} \text{ as asserted by Heinlein.}$$

73 •• Two concentric uniform thin spherical shells have masses M_1 and M_2 and radii a and $2a$, as in Figure 11-26. What is the magnitude of the gravitational force on a point particle of mass m (not shown) located (a) a distance $3a$ from the center of the shells? (b) a distance $1.9a$ from the center of the shells? (c) a distance $0.9a$ from the center of the shells?

Picture the Problem The magnitude of the gravitational force is $F_g = mg$ where g inside a spherical shell is zero and outside is given by $g = GM/r^2$.

(a) The gravitational force on a particle of mass m is given by: $F_g = mg$

At $r = 3a$, the masses of both spheres contribute to g :

$$F_g(3a) = m \frac{G(M_1 + M_2)}{(3a)^2} = \boxed{\frac{Gm(M_1 + M_2)}{9a^2}}$$

(b) At $r = 1.9a$, g due to M_2 is zero and:

$$F_g(1.9a) = m \frac{GM_1}{(1.9a)^2} = \boxed{\frac{GmM_1}{3.61a^2}}$$

(c) At $r = 0.9a$, $g = 0$ and:

$$F_g(0.9a) = \boxed{0}$$

98 •• A hole is drilled from the surface of Earth to its center as in Figure 11-30. Ignore Earth's rotation and any effects due to air resistance, and model Earth as a uniform sphere. (a) How much work is required to lift a particle of mass m from the center of Earth to Earth's surface? (b) If the particle is dropped from rest at the surface of Earth, what is its speed when it reaches the center of Earth? (c) What is the escape speed for a particle projected from the center of Earth? Express your answers in terms of m , g , and R_E .

Picture the Problem Let r represent the separation of the particle from the center of Earth and assume a uniform density for Earth. The work required to lift the particle from the center of Earth to its surface is the line integral of the gravitational force function. This function can be found from the law of gravity and by relating the mass of Earth between the particle and the center of Earth to Earth's mass. We can use the work-kinetic energy theorem to find the speed with which the particle, when released from the surface of Earth, will strike the center of Earth. Finally, the energy required for the particle to escape Earth from the center of Earth is the sum of the energy required to get it to the surface of Earth and the kinetic energy it must have to escape from the surface of Earth.

(a) Express the work required to lift the particle from the center of Earth to Earth's surface:

Using the law of gravity, express the force acting on the particle as a function of its distance from the center of Earth:

Express the ratio of M to M_E and simplify to obtain:

Substitute for M in equation (2) to obtain:

Substitute for F_g in equation (1) and evaluate the integral:

(b) Use the work-kinetic energy theorem to relate the kinetic energy of the particle as it reaches the center of Earth to the work done on it in moving it to the surface of Earth:

Substituting for W yields:

(c) Express the total energy required for the particle to escape when projected from the center of Earth:

Substituting for W yields:

Because $v_e^2 = \frac{2GM}{R_E}$:

Apply Newton's 2nd law to an object of mass m at the surface of Earth to obtain:

Substitute for GM/R_E in equation (3) to obtain:

$$W = \int_0^R \vec{F} \cdot d\vec{r} = -\int_0^R \vec{F}_g \cdot d\vec{r} = \int_0^{R_E} F_g dr \quad (1)$$

where F_g is the gravitational force acting on the particle.

$$F_g = \frac{GmM}{r^2} \quad (2)$$

where M is the mass of a sphere whose radius is r .

$$\frac{M}{M_E} = \frac{\rho \left(\frac{4}{3} \pi r^3 \right)}{\rho \left(\frac{4}{3} \pi R_E^3 \right)} = \frac{r^3}{R_E^3} \Rightarrow M = M_E \frac{r^3}{R_E^3}$$

$$F_g = \frac{GmM_E}{R_E^3} r = \frac{mgR_E^2}{R_E^3} r = \frac{mg}{R_E} r$$

$$W = \frac{mg}{R_E} \int_0^{R_E} r dr = \boxed{\frac{1}{2} gmR_E}$$

$$W = \Delta K = \frac{1}{2} mv^2$$

$$\frac{1}{2} gmR_E = \frac{1}{2} mv^2 \Rightarrow v = \boxed{\sqrt{gR_E}}$$

$$E_{\text{esc}} = W + \frac{1}{2} mv_e^2 = \frac{1}{2} mv_{\text{esc}}^2$$

where v_e is the escape speed from the surface of Earth.

$$\frac{1}{2} gmR_E + \frac{1}{2} mv_e^2 = \frac{1}{2} mv_{\text{esc}}^2$$

or, simplifying,

$$gR_E + v_e^2 = v_{\text{esc}}^2$$

$$gR_E + \frac{2GM}{R_E} = v_{\text{esc}}^2 \quad (3)$$

$$mg = \frac{Gmm}{R_E^2} \Rightarrow \frac{GM}{R_E} = gR_E$$

$$gR_E + 2gR_E = v_{\text{esc}}^2 \Rightarrow v_{\text{esc}} = \boxed{\sqrt{3gR_E}}$$

Remarks: This escape speed is approximately 122% of the escape speed from the surface of Earth.